Académie d'Orléans-Tours

Baccalauréat Général – Session 2011

Épreuve spécifique des sections européennes

Anglais / Mathématiques

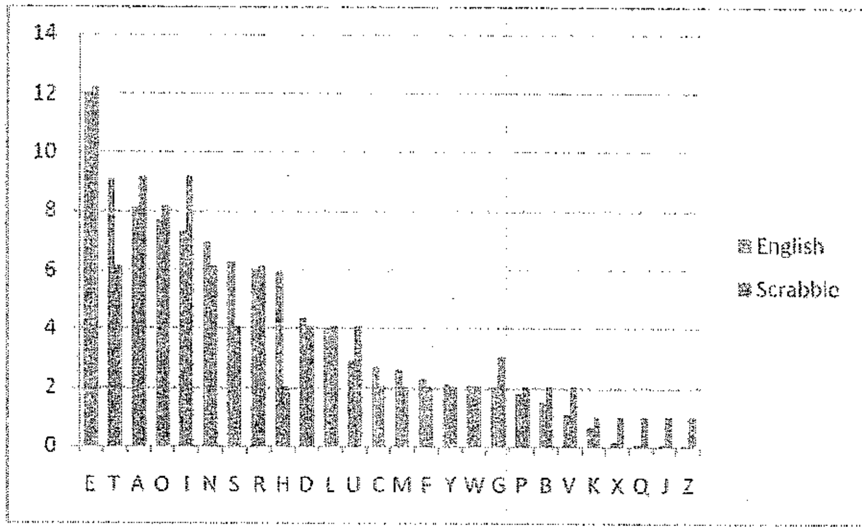
Les candidats restituent les textes à l'issue de leur épreuve.

Scrabble- corrected

1. The 5 vowels in the English alphabet are A (9 tiles), E (12 tiles), I (9 tiles), O (8 tiles) and U (4 tiles) leading to 42% chances of picking one in the bag.
2. TWELVE gives a total score of 1+4+1+1+4+1=12
3. We have 98 lettered tiles in Scrabble, so we can compute the comparison table below:

Letter	E	T	A	O	I	N	S	R	H	D	L	U	C	M	F	Y	W	G	P	B	V	K	X	Q	J	Z
English	12	9	8	8	7	7	6	6	6	4	4	3	3	3	2	2	2	2	2	1	1	1	0	0	0	0
Scrabble	12	6	9	8	9	6	4	6	2	4	4	4	2	2	2	2	2	3	2	2	2	1	1	1	1	1

In the significant proportions range (from E to M) , the Scrabble distribution overestimates the vowels share (A,O, I and U) and underestimate the use of **T**, H and S.



Newton vs Leibniz : another controversy.

Text :

The student could start noticing that Newton and Leibniz are contemporary. They were both great scientists and they both worked on some topics like differential calculus and conserved quantity during collisions. Their different approaches led to some arguments, and also the dates of publication of their works about differential calculus opened up a schism in science that would hold back British science and thinking and that would not heal for some 140 years.

The student could then develop the idea of energy conservation or energy transformation, giving some examples (crash between two cars, kinetic energy transformed into thermal energy, why not a word about nuclear energy and the Japanese catastrophe, ...).

Exercise :

1)

Height (m)	1	10	100
Impact velocity ($m.s^{-1}$)	$\simeq 4.43$	$\simeq 14$	$\simeq 44.3$

Comments : - The impact velocity is independent of the mass of the ball.
- The velocity is proportional to the square root of the height.

2)

Height (m)	1	10	100
Kinetic Energy ($Kg.m^2.s^{-2}$)	$\simeq 19.62$	$\simeq 196.2$	$\simeq 1962$

Comments : - The kinetic energy is proportional to the height. (It could be demonstrated that KE = 2gh here)
- Furthermore, for a same height, the kinetic energy is proportional to the mass of the ball and also to the square of its velocity.

3) Using Work-Energy formula $F\times d=\frac{1}{2}(mv_i^2-mv_f^2)$, it comes $F\times 0.1=196.2-0$.
Hence, F=1962 N.

Now, if the penetration in the ground is just 5 cm = 0.05 m, then we have $F\times 0.05=196.2-0$.
We deduce that $F=\frac{196.2}{0.05}=3924\,N$.

To conclude, we could say that the deeper the balls penetrates the ground, the smaller is the impact force.

At last, if the impact hole is almost 0 cm deep, then the average force of impact is infinite, because it's the result of a division by 0. In that case, the ball would more than likely explode.

Then the examiner could lead the conversation to sport mats, asking the student whether it would be best to use a soft thick mat or a hard (thin) mat in the receive area of the high jump or pole vault, and to explain the reasons why.

Which trees can you see in the forest ? CORRIGÉ

On the grid below are represented the points (m,n) in the Cartesian plane where n and m are non negative integers. We call this a *lattice* of points. Imagine that each of these points represent a tree in a well organised forest, and that you are standing at the origin O. Then you would not be able to see every tree because some would be hidden by another tree in front of it. For instance the tree B(4,2) is not visible because A(2,1) is hiding it from you. We will call A a *visible point* and B a *non visible point*.

1)The visible points are in black on the grid below.

2) n/m represents the slope (or gradient) of the line (OK). Saying that a point K is visible means that there is no other point, say L on the line segment [OK] with a smaller x-value than K. In other words, no other point L on the lattice is such that (OL) has the same slope as (OK) with a smaller x-value . That means n/m cannot be written as n'/m' with m'<m. Hence the visible points are those for which the fraction n/m is irreducible.

3)Write down all those fractions with $n \leq m \leq 5$ in increasing order:

0/1, 1/5, 1/4, 1/3, 2/5, **1/2, 3/5, 2/3, 3/4, 4/5**, 1/1

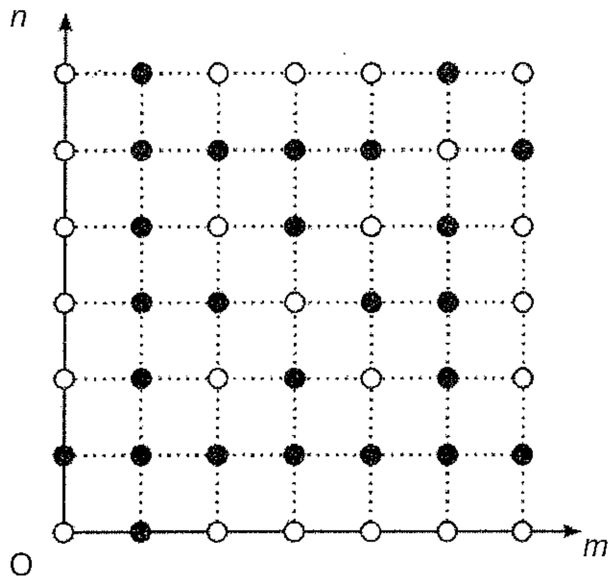
You can find these in this order this way : imagine a ray (OK) starting on the x-axis and whose slope is increasing rotating about the origin, it will hit each visible point in this order.

This is called the Farey serie of order 5. The Farey serie of order 6 is:

0/1, **1/6**, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5,**5/6**, 1/1

Note : The Farey series have the remarkable following property :

If a/b , c/d, e/f are neighbouring terms in any Farey sequence then the middle term is the *mediant* of a/b and e/f : c/d=(a+e)/(b+f). In other words, just add numerators and denominators of a/b and e/f together to find the middle term. This is equivalent to saying that for all neighbouring terms a/b , c/d: $bc - ad = 1$ which can be proved using Pick's theorem

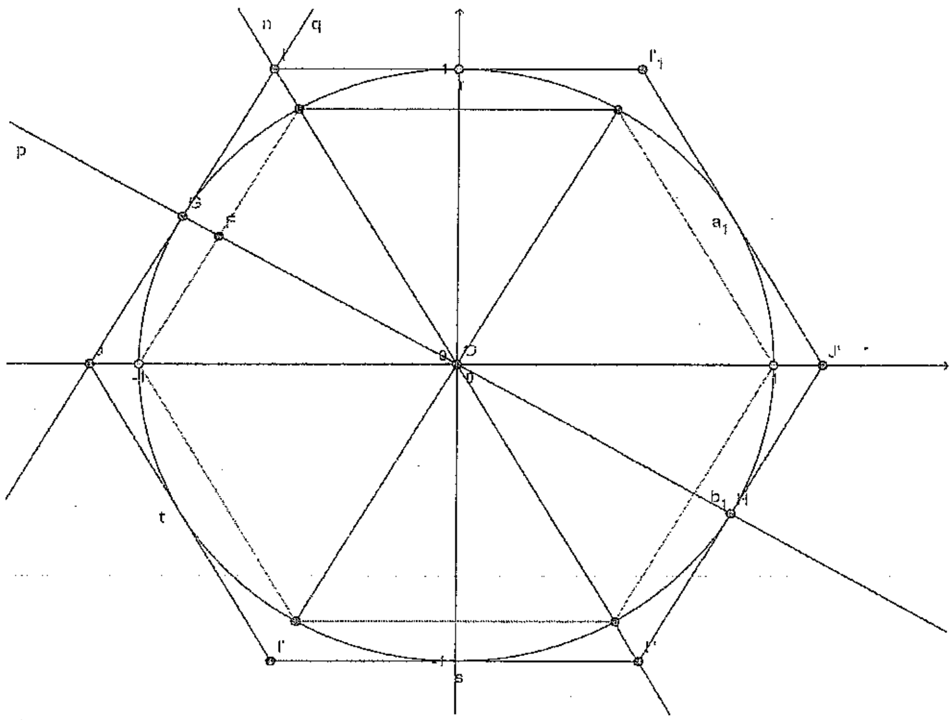


ANSWERS : *(FUNCTIONS)*

1) $V(x) = x(10 - 2x)(2 - 2x)$
2) *x is less than 5 and less than 1 so : 0*$\leq x \leq 1$
3) It depends on the choices of the pupil.
4) a) It depends on the choices of the pupil.
   b) x is closed to 0.5.

<u>Corrigé:</u>

1a) If we take n=2, we have a six-sides polygon, a hexagon.
b)



c) If we denote L as half the length of the outer hexagon's side
tan 30°= IG/OG = L/1 so L=tan 30°

d) P=12*L=12*tan(30)= 3.46......

2)

Value of n	2	3	4	5	6
Number of sides	6	....12........	....24...........	.......48.........	.....96.........
Lower bound	3,000	3,106	3,133	3,1393	3,1408
Upper bound	3.46	3,249	3,159	3,1461	3,1429
error	(3.46-3)/2-pi =0.08841	0.03591	0.00441	0.00111	0.00026

The error decreases as the number of sides increases.

Corrigé du sujet de géométrie

Have you ever heard about *sangaku* ?

I. Introduction

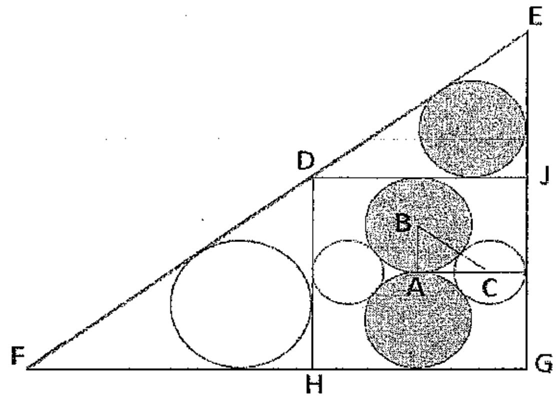
II. Questions

I. Littéralement, le mot *Sangaku* signifie *tablettes mathématiques* en japonais et qualifie les supports sur lesquels se trouvent de nombreux problèmes de mathématiques que l'on peut trouver en particulier dans les temples japonais.

II. Dans la plupart des cas, il s'agit de situations de géométrie euclidienne.

III. On trouve $4r$.

IV. Dans le triangle ABC rectangle en A, $AB^2+AC^2=BC^2$ donc $r^2+(2r-t)^2=(r+t)^2$ ce qui donne après développement et simplification $r=\frac{3t}{2}$.



V. En appliquant le théorème de Pythagore au triangle rectangle DJE, on trouve $EJ=3r$.

VI. Les trois triangles sont semblables et sont donc en particulier semblable à DEJ qui est semblable au triangle 3-4-5.

VII. Le triangle FDH est semblable au triangle DJE, donc on a : $\frac{R}{DH}=\frac{r}{3r}$ d'où $R=\frac{1}{3}DH$ soit $R=\frac{4r}{3}=2t$.

Source : http://www.cut-the-knot.org/pythagoras/Sangaku.shtml

Solutions

Part 1

1) The experiment is spinning the spinner.
2) The possible outcomes are landing on yellow, blue, green or red.
3) One event of this experiment is « landing on blue or red ».
4) The probability of landing on blue is one fourth.
5) It is equiprobable, since each outcome has a probability of one fourth.

Part 2

Experiment A:

1) The possible outcomes of this experiment are 1, 2, 3, 4, 5 and 6.
2) P(1) = 1/6 = P(2) = P(3) = P(4) = P(5) = P(6)
3) « Rolling an even number » is an event : it occurs with three outcomes « 2 », « 4 » and « 6 ».

P(even) = 3/6 = 1/2.

Experiment B:

1) P(green) = 7/24
2) P(blue) = 8/24 (= 1/3)
3) P(yellow) = 3/24 (= 1/8)
4) P(red) = 6/24 = 1/4 = 0,25
5) This is not a situation of equiprobability, because each outcome does not have the same probability.

Corrigé

a) The two common types of sequences are arithmetic and geometric.
In an arithmetic sequence, the difference between two consecutive terms is always the same. Each term is the sum of the previous one with a constant d called the common difference. In fact, an arithmetic sequence is defined by a linear function given by $a_n = d \times n + c$ and it is represented by collinear points in a coordinate plane.

In a geometric sequence, the ratio of two consecutive terms is always the same. Each term is the product of the previous one with a constant r called the common ratio. In fact, a geometric sequence is an exponential function given by $an = c \times r^n$ and it is represented by points that aren't collinear ...

Bode's sequence is neither an arithmetic sequence, nor a geometric one, because a_n is neither a linear function nor an exponential one.
$a_{n+1} - a_n = 0.3 \times 2^{n-1} + 0.4 - (0.3 \times 2^{n-2} + 0.4) = 0.3 \times (2^{n-1} - 2^{n-2}) = 0.3 \times 2^{n-2}$
so this difference is not a constant.

$$\frac{a_{n+1} - 0.4}{a_n - 0.4} = 2$$

So, $a_{n+1} = 2\,a_n - 0.4$ for $n > 1$ and the ratio between two consecutive terms can't be a constant ...

b) $a_4 = 0.3 \times 2^2 + 0.4 = 1.2 + 0.4 = 1.6$. This means that the distance between Mars and the Sun is about 1.6 x 93 million miles so Mars is 148.8 million miles far from the sun.

c) Let's add another column in the given table and fill it with the values of a_n :

Planet	Distance	n	a_n
Mercury	0.39	1	0.4
Venus	0.72	2	0.7
Earth	1.00	3	1
Mars	1.52	4	1.6
Asteroids	2.8	5	2.8
Jupiter	5.20	6	5.2
Saturn	9.54	7	10
Uranus	19.2	8	19.6
Neptune	30.1	9	38.8

For the six first values, we approximatively find exactly the actual distances but for the three last ones, there are more and more differences between the real value and the Bode's sequence term ... Bode's sequence is very precise to approximate the distance between the five nearest planets and the sun. It seems to be not precise enough for the others.

d) if we suppose that this sequence can predict the distance between the sun and a supposed ninth planet, we have to calculate a_{10} to compute it :
as $a_{10} = 77.2$ we can suppose that this planet may be 77.2 x 93 = 7179.6 millions miles far from the sun. We know that it is not Pluto because scientists have recently discovered that Pluto is only an asteroid...

Indications de correction

1.AI=(a+1)/2, étant donné que AE=a+1 et que I est le milieu de [AE].

Puis IB=AB-AI=a-(a+1)/2=(a-1)/2.

2.F appartient au cercle de centre I et passant par A. Donc IF=IA=(a+1)/2.

3.EBF est rectangle en B donc IBF l'est aussi et on peut utiliser le Théorème de Pythagore.

4.On a alors dans IBF : IF^2=IB^2+BF^2.

Soit ((a+1)/2)^2=((a-1)/2)^2+BF^2,

Puis BF^2=((a+1)/2)^2-((a-1)/2)^2=(a^2+2a+1)/4-(a^2-2a+1)/4,

Et enfin BF^2=4a/4=a, d'où BF=sqrt(a) en partant de AB=a.

5.Pour tracer un segment de longueur sqrt(5.5) :

> On reporte tout d'abord 5 fois le segment unité pour obtenir un segment de longueur 6.
>
> On trace le milieu du dernier segment unité reporté afin d'obtenir un segment [AB] de longueur 5.5 unités.
>
> On prolonge le segment [AB] d'un segment [BE] de longueur une unité.
>
> On trace le milieu I de [AE] et on construit le demi cercle supérieur de centre I passant par A.
>
> On construit la perpendiculaire à (AE) passant par B. Cette droite coupe le demi cercle en F et on a BF = sqrt(5.5).